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Inventory Model for Decaying Items with Time Varying Demand under Inflation and Trade Credit Policy

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ABSTRACT

In recent years, optimization techniques have been very helpful in managing the day-to-day real-life applications in business, manufacturing, hospitals, etc. This paper deals with the inventory model for decaying items with inflation and trade credit policy. The demand rate of the product is assumed to be time-varying and the production rate is demand-dependent. Numerical illustrations are provided to demonstrate the developed mathematical model in various cases with trade credit policies. The sensitivity analysis is also demonstrated and the convexity of the total inventory cost function for each case is revealed graphically.

KEY WORDS: Inventory, trade credit, inflation, time varying demand.

1. INTRODUCTION

The model incorporating the facility of permissible delay in payment was first discussed by Haley and Higgins (1973). Goyal (1985) explored a single item EOQ model under permissible delay in payments. In real life situations, there are products like volatile liquids, medicines, and materials, etc. in which the rate of deterioration is very large. Therefore, the loss due to deterioration should not be ignored. Aggarwal and Jaggi (1995) extended Goyal's (1985) model to allow for deteriorating items. Shortages are of great importance especially in a model that considers a delay in payment due to the fact that shortages can affect the quantity ordered to benefit from the delay in payment. Jamal *et al.* (1997) generalized the model of Aggarwal and Jaggi (1995) to allow for shortages and make it more applicable in the real world. In the above cited references, all the models were developed for a single warehouse and assumed that the available warehouse had unlimited capacity. However, this assumption is debatable in real life situations. A common practical situation is that of a limited storage space, whereas the extra storage capacity can be acquired in the form of rented warehouse if the retailer's existing storage capacity is insufficient to store the ordered quantity.

Motivated by the trade credit policy given by supplier, first time Shah and Shah (1992) developed a deterministic inventory model under the permissible delay in payment with two storage facilities. In that model, the items were considered non-deteriorating without shortage and the time horizon was infinite. Chung and Huang (2004) extended Goyal's (1985) model by considering limited storage capacity of the retailer in which the items were considered non-deteriorating. In that model, shortage was not allowed and the time horizon was infinite with constant demand rate. Ouyang *et al. (*2006) developed an inventory model for deteriorating items with permissible delay in payments. The purpose of that study was to find an optimal replenishment policy for minimizing the total relevant inventory cost. Chung and Huang (2007) presented a two-warehouse inventory model for deteriorating items under trade credit financing. In that model, shortages were not allowed and time horizon was infinite. The rate of deterioration in both warehouses was considered the same. Singh *et al.* (2008) presented a two-warehouse inventory model for deteriorating items with constant demand rate where shortages were allowed and partially backlogged. Singh and Jain (2009) proposed a deterministic inventory model with time varying deterioration rate and a linear trend in demand over a finite planning horizon. Singh and Malik (2008, 2009) proposed the mathematical model with the effect of inflation. Geetha and Uthayakumar (2010) developed an Economic Order Quantity (EOQ) model for deteriorating items with permissible delay in payments and single storage facility. In that model, shortages were partially backlogged. Malik and Sharma (2011) developed the inventory model with multivariable demand under the effect of inflation. Singh *et al.* (2014) discuss the inflation induced inventory model with stock dependent demand under the permissible delay in payment policy. Ouyang *et al.* (2015) proposed an integrated warehouse inventory model with capacity constraint and a permissible delay payment period that is order-size dependent. Tiwari *et al.* (2016) develops a two warehouse inventory model for non-instantaneous deteriorating items with permissible delay in payments under inflationary conditions. Shortages are allowed and partially backlogged, since customers' willingness to wait decreases over time. Jaggi *et al.* (2016) developed the inventory model with shortages under permissible delay in payments. Vandana and Sharma (2016) developed an inventory model for retailer's partial permissible delay-in-payment linked to order quantity with shortage, which is partial backlogged. Here, we consider two different cases, *i.e.* in first, the trade-credit period (M) is greater than or equal to the time interval, that units are depleted to zero due to demand; and later, the trade-credit period is less than the time interval. Pramanik *et al.* (2017) developed an integrated supply chain model under three level trade credit policy with price, credit period and credit amount dependent demand, where a supplier offers a credit period to his/her wholesaler to boost the demand of the item.

In the past, most work has been done by many authors under consideration negligible inflation. But in recent times many countries have been confronted with fluctuating inflation rates that often have been far from negligible level. The pioneer in this field was Buzacott (1975), who developed the first EOQ model taking inflation into account. Mishra (1979) first provided different inflation rates for various costs associated with an inventory system, under for various costs associated with an inventory system, under the assumption of constant demand. Bose *et al.* (1995) developed the EOQ inventory model under inflation and time discounting. Yang *et al.* (2001) provided an inventory models with time varying demand patterns under inflation. Yang (2004) considered a two – warehouse inventory problem for deteriorating items with constant demand rate under inflation. Chern *et al.* (2008) developed a partial backlogging inventory lot-size models for deteriorating items with fluctuating demand under inflation. Yang *et al.* (2010) proposed an inventory model under inflation for deteriorating items with stock-dependent consumption rate and partial backlogging shortages. Guria *et al.* (2013) presented with inflation and selling price dependent demand under deterministic and random planning horizons allowing and not allowing shortages. Yang and Chang (2013) incorporated a permissible delay in payment and developed a two-warehouse partial backlogging inventory model for deteriorating items with permissible delay in payment under inflation. Pal *et al.* (2015) considered a production inventory model for deteriorating items with ramp type demand rate under the effect of inflation and shortages under fuzziness. The deterioration rate is represented by a two-parameter Weibull distribution. Regarding the work in Sharma *et al.* (2013), Gupta *et al.* (2013), Malik *et al.* (2016), Vashisth *et al.* (2016), Kumar *et al.* (2016) an improved inventory model for non-instantaneous deteriorating products is obtained and demonstrated through numerical results. Time varying demand inventory model discussed by Malik *et al.* (2017), Kumar *et al.* (2017) and Malik *et al.* (2018) etc.

In this study, a production inventory model is proposed with trade-credit facility. The demand rate of the item is considered time dependent, and the production rate is assumed variable. The concept of non-instantaneous deterioration is taken into consideration as in some realistic cases deterioration of the product starts after some time. Numerous possible cases on the bases of the positions of trade-credit period and starting point of deterioration have been enlightened. Numerical examples are given to illustrate the proposed model in all cases. The sensitivity analysis is also performed and convexity of the average total cost function for each case is revealed graphically.

2. ASSUMPTIONS AND NOTATIONS

2.1. Assumptions

In developing the mathematical model of the inventory system, the following assumptions are made:

1. The inventory system involves only one item.
2. $d(t)=\theta t$ be the deterioration rate of the on-hand inventory at any time t, where $0<\theta<<1$. The deterioration starts at time $t=t_d$.
3. The deteriorated units are neither repaired nor replaced during the time period t_2.
4. The demand rate $D(t)$ is time dependent and is defined as $D(t)=\alpha+\beta t$, where parameter $\alpha>0$, $\beta\geq 0$.
5. The production rate P(t) is dependent on the demand rate, i.e., P(t) = KD(t) where K is a constant.

2.2. Notations

The following notation are used throughout the model:

$I(t)$: Inventory level at any time $t, t\geq 0$;

I_m : Maximum inventory level at the time t_1 (units);

A : Setup/ordering cost per cycle;

$P(t)$: Production rate;

t_d : Time point at which deterioration starts;

t_1 : Time point at which the production stopped;

t_2 : Production cycle time;

M : Delay period provided by the supplier to the manufacturer;

C_p : Cost per item for regular production ($);

C_0 : Production cost per item for an item produced during overtime ($);

C_h : Holding cost per unit per unit time ($);

C_d : Deterioration cost per unit per unit time ($);

S : Per unit selling price, $S>C_p$ ($);

I_e : Interest earned per unit per unit time ($);

I_c : Interest charged per unit per unit time ($);

r : Inflation rate

3. MODEL DEVELOPMENT

In the model, there are two approaches in the formulation of a production inventory system for decaying item. First, the production stopping time point is lesser than the life time of produced item and second, the production stopping time point is greater than the life time of item. To study these two occurrences, we assumed to case (1) for $(t_d>t_1)$ and (2) for $(t_1>t_d)$.

3.1 Case 1: suppose $(t_d > t_1)$

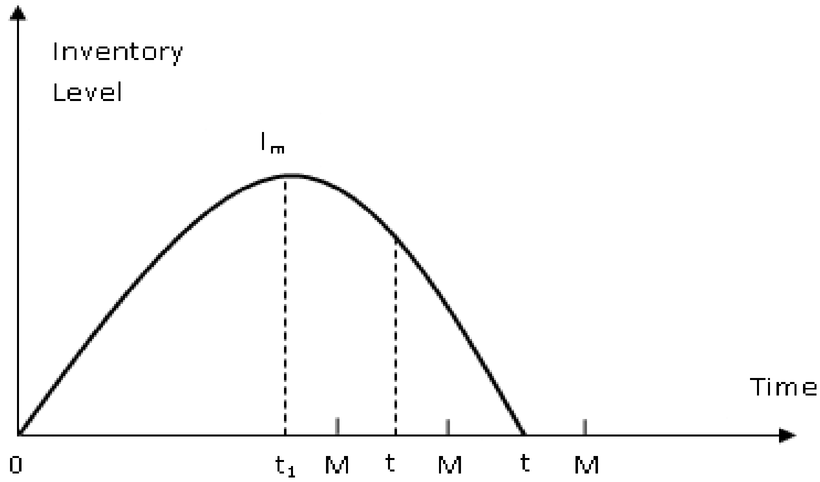

Fig.1: Inventory system when $(t_d > t_1)$.

In Case 1, the inventory system during a given cycle is depicted in Fig. 1. It is assumed that the initial stock is zero and production starts at $t = 0$. During the interval $0 \le t \le t_1$, items are produced with a production rate $P(t)$ per unit time and there are no deteriorated items during this period. At the time $t = t_1$, stock level reaches to the highest level I_m and then production is stopped. Depletion is taken place by only demand during the period $t_1 \le t \le t_d$ and by demand and deterioration both during the period $t_d \le t \le t_2$ and at $t = t_2$ the inventory becomes zero.

Following the earlier assumptions, the differential equations describing the inventory level $I(t)$ at time $t(0 \le t \le t_2)$ of this system over the production period are given by

$$\frac{dI_1(t)}{dt} = P(t) - D(t); \qquad 0 \le t \le t_1 \qquad (3.1)$$

$$\frac{dI_2(t)}{dt} = -D(t); \qquad t_1 \le t \le t_d \qquad (3.2)$$

$$\frac{dI_3(t)}{dt} + \theta t I_3(t) = -D(t); \qquad t_d \le t \le t_2 \qquad (3.3)$$

with boundary conditions $I_1(0) = 0 \, and \, I_3(t_2) = 0$.

The solutions of the above mentioned differential equations are given by

$$I_1(t) = (K-1)\left(\alpha t + \frac{\beta t^2}{2}\right) \qquad 0 \le t \le t_1 \qquad (3.4)$$

$$I_2(t) = I_m - \alpha(t - t_1) - \beta\left(\frac{t^2}{2} - \frac{t_1^2}{2}\right) \qquad t_1 \le t \le t_d \qquad (3.5)$$

$$I_3(t) = e^{\frac{-\theta t^2}{2}}\left[\alpha(t_2 - t) + \beta\left(\frac{t_2^2}{2} - \frac{t^2}{2}\right) + \alpha\theta\left(\frac{t_2^3}{6} - \frac{t^3}{6}\right) + \theta\beta\left(\frac{t_2^4}{8} - \frac{t^4}{8}\right)\right] \quad t_d \le t \le t_2 \qquad (3.6)$$

Now, the setup cost is given by

$$SC_1 = A \qquad (3.8)$$

The holding cost is given by

$$HC_1 = C_h\left[\int_0^{t_1} I_1(t)e^{-rt}dt + \int_{t_1}^{t_d} I_2(t)e^{-rt}dt + \int_{t_d}^{t_2} I_3(t)e^{-rt}dt\right]$$

$$= C_h\Bigg[(K-1)\left\{\frac{\alpha t_1^2}{2} - \frac{\alpha r t_1^3}{3} + \frac{\beta t_1^3}{6} - \frac{\beta r t_1^4}{8}\right\} + I_m(t_d - t_1) - \frac{I_m r}{2}(t_d^2 - t_1^2)$$

$$-\alpha\left(\frac{t_d^2}{2} + \frac{t_1^2}{2} - t_1 t_d - \frac{r t_d^3}{3} - \frac{r t_1^3}{6} + \frac{r t_d^2 t_1}{2}\right) - \frac{\beta}{2}\left(\frac{t_d^3}{3} + \frac{2t_1^3}{3} - t_1^2 t_d - \frac{r t_d^3}{3} - \frac{r t_1^3}{3}\right.$$

$$\left. + \frac{r t_d^2 t_1^2}{2}\right) + \alpha\left(\frac{t_d^2}{2} + \frac{t_2^2}{2} - t_2 t_d - \frac{r t_d^3}{3} - \frac{r t_2^3}{6} + \frac{r t_d^2 t_2}{2}\right) + \frac{\beta}{2}\left(\frac{t_d^3}{3} + \frac{2t_2^3}{3} - t_2^2 t_d + \frac{r t_2^4}{2} + \frac{r t_d^4}{4}\right.$$

$$\left. - \frac{r t_d^2 t_2}{2}\right) + \frac{\alpha\theta}{6}\left(\frac{t_d^4}{4} + \frac{3t_2^4}{4} - t_2^3 t_d + \frac{3r t_2^5}{10} - \frac{r t_d^5}{5} + \frac{r t_d^2 t_2^3}{2}\right) + \frac{\beta\theta}{8}\left(\frac{t_d^5}{5} + \frac{4t_2^4}{5} - t_2^2 t_d - \frac{r t_2^6}{3}\right.$$

$$\left.\left. - \frac{r t_d^6}{6} + \frac{r t_d^2 t_2^4}{2}\right)\right] \quad (3.9)$$

The deterioration cost by neglecting the higher degree terms of θ is given by

$$DC_1 = C_d \int_{t_d}^{t_2} \theta t I_3(t) e^{-rt} dt$$

$$= \theta C_d\Bigg[\alpha\left(\frac{t_2^3}{6} + \frac{t_d^3}{3} - \frac{t_2 t_d^2}{2} - \frac{r t_2^4}{12} - \frac{r t_d^4}{4} + \frac{r t_2 t_d^3}{3} - \frac{\theta t_2^5}{40} - \frac{\theta t_d^5}{10}\right.$$

$$\left. + \frac{\theta t_2 t_d^4}{8} + \frac{\theta r t_2^6}{60} + \frac{\theta r t_d^6}{12} - \frac{\theta r t_2 t_d^5}{10}\right) + \frac{\beta}{2}\left(\frac{t_2^4}{2} + \frac{t_d^4}{4} - \frac{t_2^2 t_d^2}{2} - \frac{2r t_2^5}{15}\right.$$

$$\left. - \frac{r t_d^5}{5} + \frac{r t_2^2 t_d^3}{3} - \frac{\theta t_2^6}{24} - \frac{\theta t_d^6}{12} + \frac{\theta t_2^2 t_d^4}{8} + \frac{\theta r t_2^7}{35} + \frac{\theta r t_d^7}{14} - \frac{\theta r t_2^2 t_d^5}{10}\right) +$$

$$\frac{\alpha\theta}{6}\left(\frac{3t_2^5}{10} + \frac{t_d^5}{5} - \frac{t_2^3 t_d^2}{2} - \frac{r t_2^6}{6} - \frac{r t_d^6}{6} + \frac{r t_2^3 t_d^3}{3} - \frac{3\theta t_2^7}{14} - \frac{\theta t_d^7}{14} + \frac{\theta t_2^3 t_d^4}{8} + \right.$$

$$\left. \frac{3\theta r t_2^8}{80} + \frac{\theta r t_d^8}{16} - \frac{\theta r t_2^3 t_d^5}{10}\right) + \frac{\beta\theta}{8}\left(\frac{t_2^6}{3} + \frac{t_d^6}{6} - \frac{t_2^4 t_d^2}{2} - \frac{4r t_2^7}{21} - \frac{r t_d^7}{7} + \frac{r t_2^4 t_d^3}{3}\right.$$

$$\left.\left. - \frac{\theta t_2^8}{16} - \frac{\theta t_d^8}{16} + \frac{\theta t_2^4 t_d^4}{8} + \frac{2\theta r t_2^9}{45} + \frac{\theta r t_d^9}{18} - \frac{\theta r t_2^4 t_d^5}{10}\right)\right] \quad (3.10)$$

The production cost is given by

$$PC_1 = C_p\left(\int_0^{t_1} a e^{-rt} dt\right) + C_0 \int_0^{t_1} bK(\alpha + \beta t) e^{-rt} dt$$

$$= C_p\left(a t_1 - \frac{a r t_1^2}{2}\right) + C_0 bK\left(\alpha t_1 + \frac{\beta t_1^2}{2} - \frac{\alpha r t_1^2}{2} - \frac{\beta r t_1^3}{3}\right) \quad (3.11)$$

Now, permissible delay period M is assumed greater than the production run time t_1. So, in Case 1 there may arise three cases according to the position of delay period M such as case (i) $t_1 \le M < t_d < t_2$, case (ii) $t_1 < t_d \le M < t_2$ and case (iii) $M > t_2$. Now, we will discuss these cases separately one by one.

3.1.1 Special Case (i): When $t_1 \le M < t_d < t_2$

In this case, interest earned is given by

$$IE_{11} = SI_e \int_0^M (\alpha + \beta t) te^{-rt} dt$$
$$= SI_e \left[\frac{\alpha M^2}{2} + \frac{\beta M^3}{3} - \frac{\alpha r M^3}{3} - \frac{\beta r M^4}{4} \right] \quad (3.12)$$

Interest charged is given by

$$IC_{11} = C_p I_c \int_M^{t_2} (\alpha + \beta t) te^{-rt} dt$$
$$= C_p I_c \left[\frac{\alpha}{2}(t_2^2 - M^2) + \frac{\beta}{3}(t_2^3 - M^3) - \frac{\alpha r}{3}(t_2^3 - M^3) - \frac{\beta r}{4}(t_2^4 - M^4) \right] \quad (3.13)$$

Now, the average total cost per unit time of inventory system in special case (i) of case 1 is given by

$$ATC_{11}(t_2) = \frac{1}{T}[SC_1 + HC_1 + DC_1 + PC_1 + IC_{11} - IE_{11}] \quad (3.14)$$

3.1.1.1 Solution procedure

The objective of this study is to minimize the average total cost per unit time. The optimal value of t_2 can be determined by solving the equation $dATC_{11}(t_2)/dt_2 = 0$ with the help of software Mathematica 8.0 and the minimum average total cost can be obtained from (3.14).

3.1.2 Special Case (ii): When $t_1 < t_d \le M < t_2$

In this case, interest earned, interest charged are similar to the case (i).

Now, the average total cost per unit time of inventory system in special case (ii) is given by

$$ATC_{12}(t_2) = \frac{1}{T}[SC_1 + HC_1 + DC_1 + PC_1 + IC_{11} - IE_{11}] \quad (3.15)$$

Solution procedure given in case (i) can be used to find the optimal solution.

3.1.3 Special Case (iii): When $M > t_2$

In this case, interest earned is given by

$$IE_{12} = SI_e \left[\int_0^{t_2} (\alpha + \beta t) te^{-rt} dt + (M - t_2) \int_0^{t_2} (\alpha + \beta t) e^{-rt} dt \right]$$
$$= SI_e \left[M \left(\alpha t_2 + \frac{\beta t_2^2}{2} - \frac{\alpha r t_2^2}{2} - \frac{\beta r t_2^3}{3} \right) - \left(\frac{\alpha t_2^2}{2} + \frac{\beta t_2^3}{6} - \frac{\alpha r t_2^3}{6} - \frac{\beta r t_2^4}{12} \right) \right] \quad (3.16)$$

There is no interest charged for this case, so we have

$$IC_{12} = 0 \quad (3.17)$$

Now, the average total cost per unit time of inventory system in special case (iii) of case 1 is given by

$$ATC_{13}(t_2) = \frac{1}{T}[SC_1 + HC_1 + DC_1 + PC_1 + IC_{12} - IE_{12}] \quad (3.18)$$

3.2 Case 2: suppose $(t_d < t_1)$

In Case 2, the inventory system during a given cycle is depicted in Fig. 2.

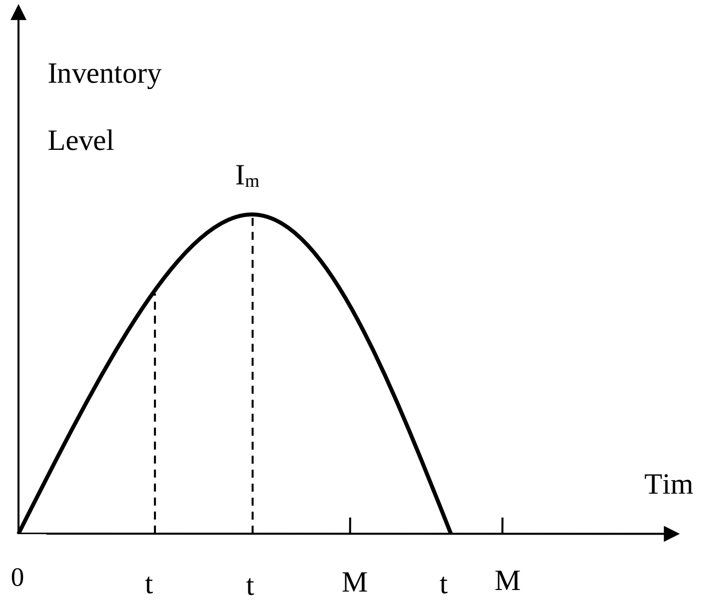

Fig.2: Inventory system when $(t_d < t_1)$.

It is assumed that the initial stock is zero and just after $t=0$, production starts. During the interval $0 \le t \le t_d$, items are produced at the rate $p(t)$ units per unit time and there are no deteriorated items. At the time $t=t_d$, the deterioration starts. At the time $t=t_1$, stock level reaches to the highest level I_m and then production is stopped. Depletion is started by demand and deterioration both during the period $t_1 \le t \le t_2$ and inventory becomes zero at $t=t_2$.

The differential equations describing the inventory level $I(t)$ at time $t(0 \le t \le t_2)$ of the inventory system in Case 2 over the production period are given by

$$\frac{dI_1(t)}{dt} = p(t) - D(t); \qquad 0 \le t \le t_d \qquad (3.19)$$

$$\frac{dI_2(t)}{dt} + d(t) I_2(t) = P(t) - D(t); \qquad t_d \le t \le t_1 \qquad (3.20)$$

$$\frac{dI_3(t)}{dt} + d(t) I_3(t) = -D(t); \qquad t_1 \le t \le t_2 \qquad (3.21)$$

with boundary conditions $I_1(0)=0, I_3(t_2)=0$.

The solutions of the above mentioned differential equations are given as respectively

$$I_1(t) = (K-1)\left(\alpha t + \frac{\beta t^2}{2}\right) \qquad 0 \le t \le t_d \qquad (3.22)$$

$$I_2(t) = I_m + (K-1)\left[\alpha(t-t_1) + \frac{\beta}{2}(t^2 - t_1^2) + \frac{\alpha\theta}{6}(t^3 - t_1^3) + \frac{\beta\theta}{8}(t^4 - t_1^4)\right] e^{-\theta t^2 / 2}$$

$$t_d \le t \le t_1 \qquad (3.23)$$

$$I_3(t) = e^{\frac{-\theta t^2}{2}}\left[\alpha(t_2 - t) + \beta\left(\frac{t_2^2}{2} - \frac{t^2}{2}\right) + \alpha\theta\left(\frac{t_2^3}{6} - \frac{t^3}{6}\right) + \theta\beta\left(\frac{t_2^4}{8} - \frac{t^4}{8}\right)\right] \quad t_1 \le t \le t_2 \qquad (3.24)$$

Now, the holding cost is given by

$$HC_2 = C_h\left[\int_0^{t_d} I_1(t) e^{-rt} dt + \int_{t_d}^{t_1} I_2(t) e^{-rt} dt + \int_{t_1}^{t_2} I_3(t) e^{-rt} dt\right]$$

$$=C_h\left[I_m(t_1-t_d)-\frac{rI_m}{2}(t_1^2-t_d^2)+(K-1)\left\{\alpha\left(t_1t_d-\frac{t_1^2}{2}+\frac{rt_1^3}{6}-rt_dt_1\right)+\right.\right.$$
$$+\frac{\beta}{2}\left(t_1^2t_d-\frac{2t_1^3}{3}+\frac{rt_1^4}{2}-rt_d^2t_1^2\right)+\frac{\alpha\theta}{6}\left(t_1^3t_d-\frac{3t_1^4}{4}-\frac{t_d^4}{4}+\frac{3rt_1^5}{10}+\frac{rt_d^5}{5}-\frac{rt_d^2t_1^3}{3}\right)$$
$$+\frac{\beta\theta}{8}\left(t_1^4t_d-\frac{4t_1^5}{5}-\frac{t_d^5}{5}+\frac{rt_1^6}{3}+\frac{rt_d^6}{6}-\frac{rt_d^2t_1^4}{2}\right)\Bigg\}+\alpha\left(\frac{t_d^2}{2}+\frac{t_2^2}{2}-t_2t_d-\frac{rt_d^3}{3}-\frac{rt_2^3}{6}+\frac{rt_d^2t_2}{2}\right)$$
$$+\frac{\beta}{2}\left(\frac{t_d^3}{3}+\frac{2t_2^3}{3}-t_2^2t_d+\frac{rt_2^4}{2}+\frac{rt_d^4}{4}-\frac{rt_d^2t_2}{2}\right)+\frac{\alpha\theta}{6}\left(\frac{t_d^4}{4}+\frac{3t_2^4}{4}-t_2^3t_d+\frac{3rt_2^5}{10}-\frac{rt_d^5}{5}+\frac{rt_d^2t_2^3}{2}\right)$$
$$\left.+\frac{\beta\theta}{8}\left(\frac{t_d^5}{5}+\frac{4t_2^4}{5}-t_2^2t_d-\frac{rt_2^6}{3}-\frac{rt_d^6}{6}+\frac{rt_d^2t_2^4}{2}\right)\right]\tag{3.25}$$

The deterioration cost by neglecting the higher degree terms of θ is given by

$$DC_2=C_d\left[\int_{t_d}^{t_1}\theta tI_2(t)e^{-rt}dt+\int_{t_1}^{t_2}\theta tI_3(t)e^{-rt}dt\right]$$
$$=\theta C_d\left[I_m\left\{\frac{1}{2}(t_1^2-t_d^2)-\frac{r\theta}{3}(t_1^3-t_d^3)\right\}+(K-1)\left\{\alpha\left(\frac{\theta t_1t_d^2}{2}-\frac{\theta t_1^3}{6}\right.\right.\right.$$
$$-\frac{\theta t_d^3}{3}+\frac{\theta^2t_1^5}{40}+\frac{\theta^2t_d^5}{10}-\frac{\theta^2t_d^4t_1}{8}+\frac{r\theta t_1^4}{12}+\frac{r\theta t_d^4}{4}-\frac{r\theta t_1t_d^3}{3}-\frac{r\theta^2t_1^6}{60}-\frac{r\theta^2t_d^6}{12}+\frac{r\theta^2t_d^5t_1}{10}\Bigg)$$
$$+\frac{\beta}{2}\left(\frac{\theta t_1^2t_d^2}{2}-\frac{\theta t_1^4}{4}-\frac{\theta t_d^4}{4}+\frac{\theta^2t_1^6}{24}+\frac{\theta^2t_d^6}{12}-\frac{\theta^2t_d^4t_1^2}{8}+\frac{2r\theta t_1^5}{15}+\frac{r\theta t_d^5}{5}-\frac{r\theta t_1^2t_d^3}{3}-\frac{2r\theta^2t_1^7}{70}\right.$$
$$\left.-\frac{r\theta^2t_d^7}{14}+\frac{r\theta^2t_d^5t_1^2}{10}\right)+\frac{\alpha\theta}{6}\left(\frac{\theta t_1^3t_d^2}{2}-\frac{3\theta t_1^5}{10}-\frac{\theta t_d^5}{5}+\frac{3\theta^2t_1^7}{56}+\frac{\theta^2t_d^7}{14}-\frac{\theta^2t_d^4t_1^3}{8}+\frac{r\theta t_1^6}{6}+\right.$$
$$\left.\frac{r\theta t_d^6}{6}-\frac{r\theta t_1^3t_d^3}{3}-\frac{3r\theta^2t_1^8}{80}-\frac{r\theta^2t_d^8}{16}+\frac{r\theta^2t_d^5t_1^3}{10}\right)+\frac{\beta\theta}{8}\left(\frac{\theta t_1^4t_d^2}{2}-\frac{\theta t_1^6}{3}-\frac{\theta t_d^6}{6}+\frac{\theta^2t_1^8}{16}+\frac{\theta^2t_d^8}{16}-\right.$$
$$\left.\frac{\theta^2t_d^4t_1^4}{8}+\frac{4r\theta t_1^7}{21}+\frac{r\theta t_d^7}{7}-\frac{r\theta t_1^4t_d^3}{3}-\frac{2r\theta^2t_1^9}{45}-\frac{r\theta^2t_d^9}{18}+\frac{r\theta^2t_d^5t_1^4}{10}\right)\Bigg\}+\left\{\alpha\left(-\frac{\theta t_2t_1^2}{2}+\frac{\theta t_2^3}{6}+\right.\right.$$
$$\left.+\frac{\theta t_1^3}{3}-\frac{\theta^2t_2^5}{40}-\frac{\theta^2t_1^5}{10}+\frac{\theta^2t_1^4t_2}{8}-\frac{r\theta t_2^4}{12}-\frac{r\theta t_1^4}{4}+\frac{r\theta t_2t_1^3}{3}+\frac{r\theta^2t_2^6}{60}+\frac{r\theta^2t_1^6}{12}-\frac{r\theta^2t_1^5t_2}{10}\right)+$$
$$\frac{\beta}{2}\left(-\frac{\theta t_1^2t_2^2}{2}+\frac{\theta t_1^4}{4}+\frac{\theta t_2^4}{4}-\frac{\theta^2t_2^6}{24}-\frac{\theta^2t_1^6}{12}+\frac{\theta^2t_1^4t_2^2}{8}-\frac{2r\theta t_2^5}{15}-\frac{r\theta t_1^5}{5}+\frac{r\theta t_2^2t_1^3}{3}+\frac{2r\theta^2t_2^7}{70}\right.$$
$$\left.+\frac{r\theta^2t_1^7}{14}+\frac{r\theta^2t_d^5t_1}{10}\right)+\frac{\alpha\theta}{6}\left(-\frac{\theta t_2^3t_1^2}{2}+\frac{3\theta t_2^5}{10}+\frac{\theta t_1^5}{5}-\frac{3\theta^2t_1^7}{56}-\frac{\theta^2t_1^7}{14}+\frac{\theta^2t_1^4t_2^3}{8}-\frac{r\theta t_2^6}{6}\right.$$
$$\left.-\frac{r\theta t_1^6}{6}+\frac{r\theta t_1^3t_2^3}{3}+\frac{3r\theta^2t_2^8}{80}+\frac{r\theta^2t_1^8}{16}-\frac{r\theta^2t_1^5t_2^3}{10}\right)+\frac{\beta\theta}{8}\left(-\frac{\theta t_2^4t_1^2}{2}+\frac{\theta t_1^6}{3}+\frac{\theta t_2^6}{6}-\frac{\theta^2t_2^8}{16}-\right.$$
$$\left.\left.\left.\frac{\theta^2t_1^8}{16}+\frac{\theta^2t_1^4t_2^4}{8}-\frac{4r\theta t_2^7}{21}-\frac{r\theta t_1^7}{7}+\frac{r\theta t_2^4t_1^3}{3}+\frac{2r\theta^2t_2^9}{45}+\frac{r\theta^2t_1^9}{18}-\frac{r\theta^2t_1^5t_2^4}{10}\right)\right\}\right]\tag{3.26}$$

The production cost is given by

$$PC_2 = C_p\left(\int_0^{t_1} ae^{-rt}\,dt\right) + C_0\left(\int_0^{t_1} b(\alpha+\beta t)e^{-rt}\,dt\right)$$
$$= C_p\left(at_1 - \frac{art_1^2}{2}\right) + C_0 bK\left(\alpha t_1 + \frac{\beta t_1^2}{2} - \frac{\alpha r t_1^2}{2} - \frac{\beta r t_1^3}{3}\right) \quad (3.27)$$

The setup cost for this case is same as in case 1.
Now, in case 2 there may arise two cases according to the position of delay period M such as special case (i) $t_1 < M \le t_2$ and special case (ii) $M > t_2$. Now, we will discuss these cases separately one by one.

3.2.1 Special Case (i): When $t_1 < M \le t_2$
In this case, interest earned, interest charged are similar to the special case (i) of Case 1.

Now, the average total cost per unit time of inventory system in special case (i) of Case 2 is given by

$$ATC_{21}(t_2) = \frac{1}{T}[SC_1 + HC_2 + DC_2 + PC_2 + IC_{11} - IE_{11}] \quad (3.28)$$

3.2.2 Special Case (ii): When $M > t_2$
In this case, interest earned, interest charged are similar to the special case (iii) of Case1.
Now, the average total cost per unit time of inventory system in special case (ii) of Case 2 is given by

$$ATC_{22}(t_2) = \frac{1}{T}[SC_1 + HC_2 + DC_2 + PC_2 + IC_{12} - IE_{12}] \quad (3.29)$$

4. NUMERICAL EXAMPLE

For illustration of all special cases of the two Cases discussed above in developed model, some numerical examples are presented. The figures have been taken randomly from literatures in appropriate units.

Example 1: (Special Case (i) of Case 1) The input values of the system parameters are taken randomly as follows:

$A = 200, t_d = 0.25, \theta = 0.01, M = 0.20, \alpha = 100, \beta = 0.05, C_p = 5, C_h = 2.8, C_d = 3, S = 10, I_e = 0.08, I_c = 0.09$

Then, the optimal solution is: $t_1^* = 0.173, t_2^* = 0.374,\ ATC_{11}^*(t_2^*) = 328.162$ and the convexity of the average total cost function is shown graphically in Fig. 3.

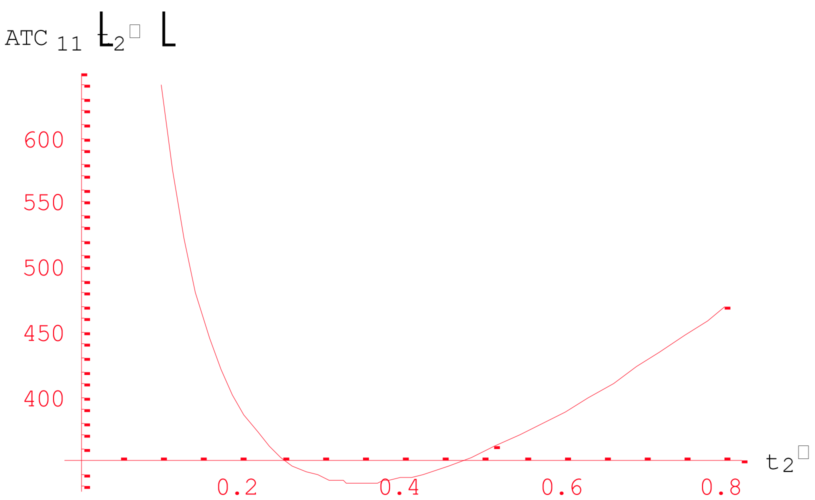

Fig. 3: Convexity of the average total cost function for Special Case (i) of Case 1

Example 2: (Special Case (ii) of Case 1) The input values of the system parameters are taken as follows:

$A = 200, t_d = 0.25, \theta = 0.01, M = 0.30, \alpha = 100, \beta = 0.05, C_p = 5, C_h = 2.8, C_d = 3, S = 10, I_e = 0.08, I_c = 0.09$

Then, the optimal solution is: $t_1^* = 0.216, t_2^* = 0.415,$

$ATC_{12}^*(t_2^*) = 388.217$ and the convexity of the average total cost function is shown graphically in Fig. 4.

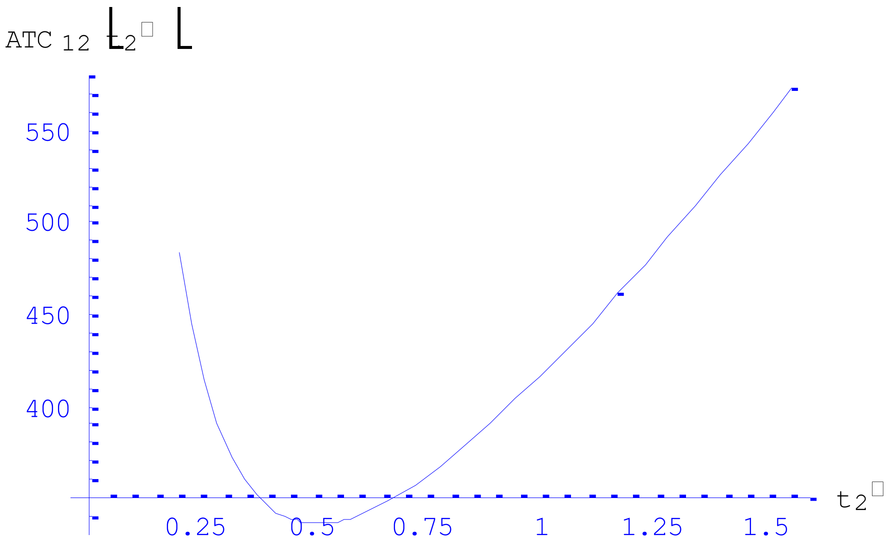

Fig. 4: Convexity of the average total cost function for Special Case (ii) of Case 1

Example 3: (Special Case (iii) of Case 1) The input values of the system parameters are taken as follows: $A = 200, t_d = 0.25, \theta = 0.01, M = 0.40, \alpha = 100, \beta = 0.05, C_p = 5, C_h = 2.8, C_d = 3, S = 10, I_e = 0.08, I_c = 0.09$

Then, the optimal solution is: $t_1^* = 0.258, t_2^* = 0.461,\quad ATC_{13}^*(t_2^*) = 418.248$ and the convexity of the average total cost function is shown graphically in Fig. 5.

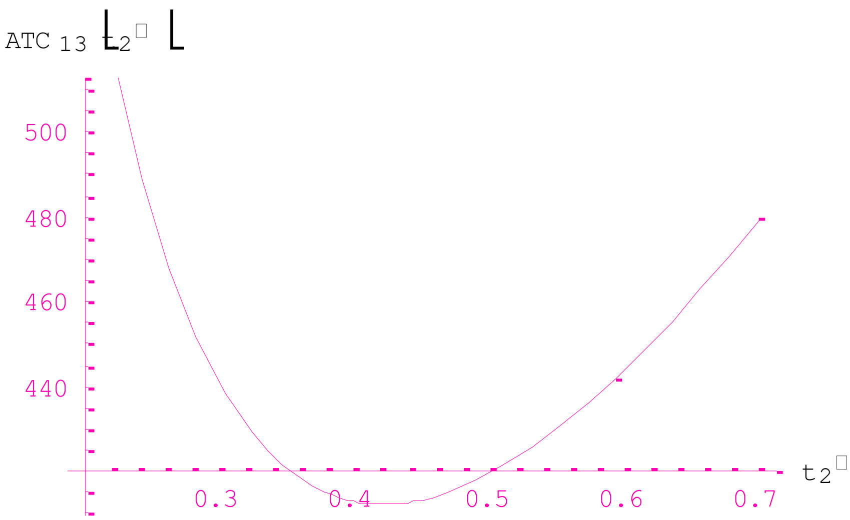

Fig. 5: Convexity of the average total cost function for Special Case (iii) of Case 1

Example 4: (Special Case (i) of Case 2) The input values of the system parameters are taken as follows: $A = 250, t_d = 0.20, \theta = 0.01, M = 0.10, \alpha = 80, \beta = 0.08, C_p = 4, C_h = 2.5, C_d = 2, S = 8, I_e = 0.08, I_c = 0.09$

Then, the optimal solution is: $t_1^* = 0.128, t_2^* = 0.246,\quad ATC_{21}^*(t_2^*) = 238.607$ and the convexity of the average total cost function is shown graphically in Fig. 6.

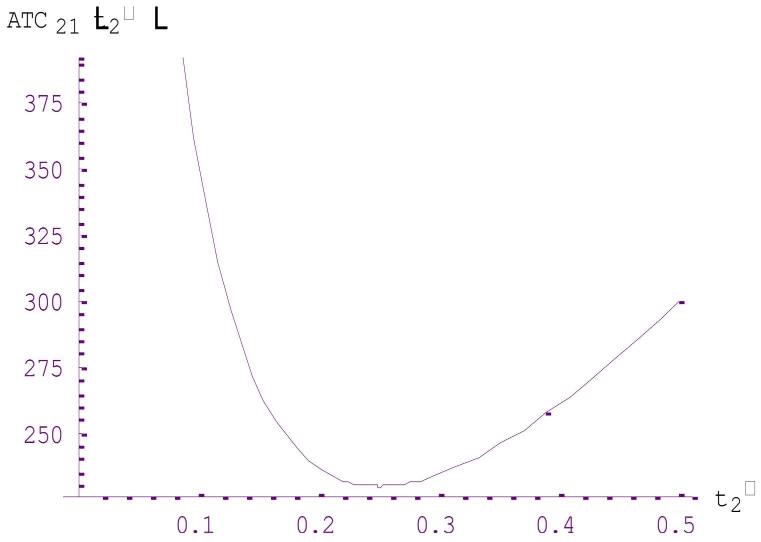

Fig. 6: Convexity of the average total cost function for Special Case (i) of Case 2

Example 5: (Special Case (ii) of Case 2) The input values of the system parameters are taken as follows: $A = 250, t_d = 0.20, \theta = 0.01, M = 0.20, \alpha = 80, \beta = 0.08, C_p = 4, C_h = 2.5, C_d = 2, S = 8, I_e = 0.08, I_c = 0.09$

Then, the optimal solution is: $t_1^* = 0.164, t_2^* = 0.281,\quad ATC_{22}^*(t_2^*) = 278.381$ and the convexity of the average total cost function is shown graphically in Fig. 7.

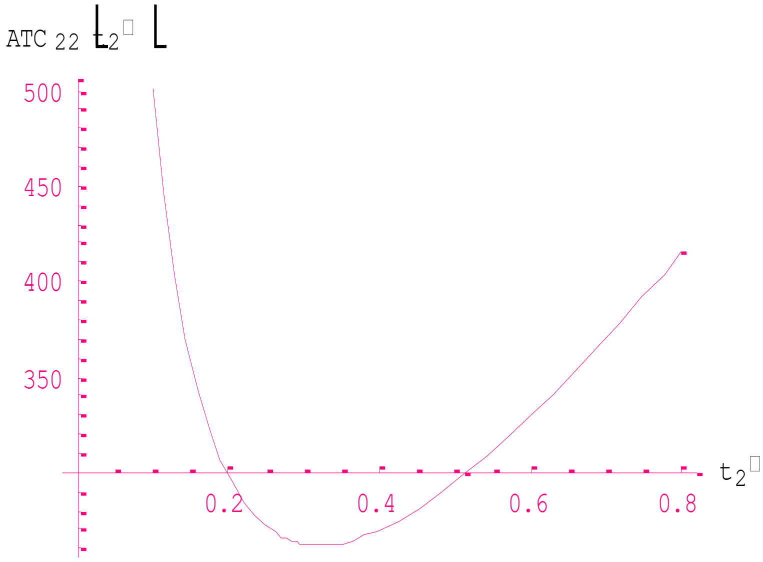

Fig. 7: Convexity of the average total cost function for Special Case (ii) of Case 2

5. Sensitivity Analysis

In Special case (i) of Case 1 when $(t_1 < t_d)$, to discuss the effect of changes of the model parameters $S, t_d, \alpha, \beta, C_h$ *and* C_p on the optimal value of the average total cost $ATC_{11}(t_2^*)$, the production stopping time point (t_1^*), and the cycle time (t_2^*) we have discussed the sensitivity analysis in this section. The different values of these parameter according to $\pm 5\%$ and $\pm 10\%$ change have been taken and the effect on $ATC_{11}(t_2^*)$, t_1^*, and t_2^* are presented in the following Table 1. In the similar manner sensitivity analysis can be performed for other cases.

Table 1: Sensitivity Analysis for Special Case (i) of Case 1

Parameters	**% Change in Parameter**	**%Change in the value of**		
		t_1^*	t_2^*	$ATC_{11}(t_2^*)$
S	-20%	+0.37	+0.89	-4.81
	-10%	+0.19	+0.58	-2.31
	10%	-0.18	-0.55	+2.20
	20%	-0.38	-0.83	+4.73
t_d	-20%	+4.12	+0.10	-0.46
	-10%	+2.91	+0.08	-0.24
	10%	-3.13	-0.05	+0.27
	20%	-5.24	-0.07	+0.43
α	-20%	+2.62	+0.15	+9.40
	-10%	+1.23	+0.07	+4.69
	10%	-1.53	-0.08	-4.51
	20%	-3.14	-0.16	-9.29
β	-20%	+0.18	-0.08	+0.86
	-10%	+0.11	-0.05	+0.22
	10%	-0.09	+0.05	-0.20
	20%	-0.13	+0.07	-0.83
C_h	-20%	+0.34	+0.52	+1.52
	-10%	+0.23	+0.40	+0.78
	10%	-0.22	-0.41	-0.92
	20%	-0.35	-0.53	-1.58
C_p	-20%	-0.07	-1.34	-4.67
	-10%	-0.04	-0.61	-2.27
	10%	+0.05	+0.72	+2.29
	20%	+0.08	+1.29	+4.71

Some important observations drawn from Table 1 are given as follows:

1. It is observed that from Table 1 as the selling price 'S' increases the average total cost per unit time decreases, because increase in selling price increase the sales revenue and interest earned so the on the whole cost of the system decreases.
2. It is observed from Table 1 that as 't_d' increases the average total cost of the system slightly decreases which is evident.
3. It is observed from Table 1 as the demand factors 'α' and 'β' increases the average total cost per unit time increases. The reason behind that more demand implies more production so the total average cost increases.
4. As the holding cost parameter 'C_h' and regular production parameter 'C_p' increase the average total cost of the system increases which is realistic.

6. CONCLUSION

In this article, a production inventory model is developed in which production rate is taken as demand dependent. It is assumed that demand rate is a linearly increasing function of time and after a influenced period deterioration occurs with a variable rate. In this model we assumed the concept of permissible delay to make the model more practical. Different cases of trade-credit have been discussed and illustrated with some numerical examples. Sensitivity analysis is also implemented and obtained results shows that the model is quite stable and applicable in real market. A further research can be done by allowing the model for progressive permissible delay in payment.

REFERENCES

[1]. Aggarwal, S.P., Jaggi Aggarwal, S.P., Jaggi, C.K. (1995) Ordering policies of deteriorating items under permissible delay in payments, *Journal of the Operational Research Society*, 46, 658-662.

[2]. Buzacott, J. (1975) Economic order quantities with inflation, *Operations Research Quarterly* 26(3), 553-558.

[3]. Chern, M.S., Yang, H.L., Teng, J.T., Papachristos, S. (2008) Partial backlogging inventory lot-size models for deteriorating items with fluctuating demand under inflation, *European Journal of Production Economics* 191, 127-141.

[4]. Chuang, K.J., Huang, T.S. (2007) The optimal retailer's ordering policies for deteriorating items with limited storage capacity under trade credit financing, *International Journal of Production Economics*, 106, 127-145.

[5]. Chuang, K.J., Huang, Y.F. (2004) Optimal replenishment policies for EOQ inventory model with limited storage capacity under permissible delay in payments, *Opsearch*, 41, 16-34.

[6]. Geetha, K.V., Uthayakumar, R. (2010) Economic design of an inventory policy for non-instantaneous deteriorating items under permissible delay in payments, *Journal of Computational and Applied Mathematics*, 233(10), 2492-2505.

[7]. Goyal, S.K. (1985) Economic order quantity under conditions of permissible delay in payments, *Journal of the Operational Research Society*, 36, 335-338.

[8]. Gupta, K. K., Sharma, A., Singh, P. R., Malik, A. K. (2013). Optimal ordering policy for stock-dependent demand inventory model with non-instantaneous deteriorating items. International Journal of Soft Computing and Engineering, 3(1), 279-281.

[9]. Guria, A., Das, B., Mondal, S., Maiti, M. (2013) Inventory policy for an item with inflation induced purchasing price, selling price and demand with immediate part payment, Applied Mathematical Modelling, 37(1-2), 240-257.

[10]. Haley, C.W., Higgins, H.C. (1973) Inventory policy and trade credit financing, Management Science, 20(4), 464-471.

[11]. Jamal, A.M.M., Sarker, B.R., Wang, S. (1997) An ordering policy for deteriorating items with allowable shortages and permissible delay in payment, *Journal of the Operational Research Society*, 48, 826-833.

[12]. Jaggi, CK., Yadavalli, V.S.S., Verma, M., Sharma, A. (2015) An EOQ model with allowable shortage under trade credit in different scenario, Applied Mathematics and Computation, 252, 541-551.

[13]. Kumar, S., Malik, A. K., Sharma, A., Yadav, S. K., Singh, Y. (2016, March). An inventory model with linear holding cost and stock-dependent demand for non-instantaneous deteriorating items. In AIP Conference Proceedings (Vol. 1715, No. 1, p. 020058). AIP Publishing LLC.

[14]. Kumar, S., Singh, Y., Malik, A. K. (2017). An inventory model for both variable holding and sales revenue cost, Asian Journal of Management, 8(4), 1111-1114.

[15]. Malik, A.K. and Sharma, A. (2011). An Inventory Model for Deteriorating Items with Multi-Variate Demand and Partial Backlogging Under Inflation, International Journal of Mathematical Sciences, 10(3-4), 315-321.

[16]. Malik, A. K., Shekhar, C., Vashisth, V., Chaudhary, A. K., Singh, S. R. (2016, March). Sensitivity analysis of an inventory model with non-instantaneous and time-varying deteriorating Items. In AIP Conference Proceedings (Vol. 1715, No. 1, p. 020059). AIP Publishing LLC.

[17]. Malik, A. K., Chakraborty, D., Kumar, S. (2017). Quadratic Demand based Inventory Model with Shortages and Two Storage Capacities System. Research Journal of Engineering and Technology, 8(3), 213-218.

[18]. Malik, A. K., Vedi, P., Kumar, S. (2018). An inventory model with time varying demand for non-instantaneous deteriorating items with maximum life time. International Journal of Applied Engineering Research, 13(9), 7162-7167.

[19]. Mishra, R.B. (1979) A note on optimal inventory management under inflation, Naval Research Logistics Quarterly 26(2), 161-165.

[20]. Ouyang, L.Y., Wu, K.S., Yang, C.T. (2006) A study on an inventory model for non-instantaneous deteriorating items with permissible delay in payments, Computers & Industrial Engineering, 51(4), 637-651.

[21]. Ouyang, L.Y., Ho, C.H., Su, C.H., Yang, C.T. (2015) An integrated inventory model with capacity constraint and order-size dependent trade credit, Computers & Industrial Engineering, 84, 133-143.

[22]. Pramanik, P., Maiti, M.K., Maiti, M. (2017) A supply chain with variable demand under three level trade credit policy, Computers & Industrial Engineering, 106, 205-221.
[23]. Shah, N.H., Shah, Y.K. (1992) Deterministic lot-size inventory model when delay in payments is permissible for a system with two storage facilities, I. E. Journal, XX, 3, 121-128.
[24]. Sharma, A., Gupta, K. K., Malik, A. K. (2013). Non-Instantaneous Deterioration Inventory Model with inflation and stock-dependent demand. International Journal of Computer Applications, 67(25), 6-9.
[25]. Singh, S. R. and Malik, A. K. (2008). Effect of inflation on two warehouse production inventory systems with exponential demand and variable deterioration. International Journal of Mathematical and Applications, 2(1-2), 141-149.
[26]. Singh, S.R., Kumar, N., Kumari, R. (2008) Two-Warehouse inventory model for deteriorating items with partial backlogging under the conditions of permissible delay in payments, International Transactions in Mathematical Sciences & Computer, 1 (1), 123-134.
[27]. Singh, S.R., Jain, R. (2009) On reserve money for an EOQ model in an inflationary environment under supplier credits, Opsearch, 46 (3), 303-320.
[28]. Singh, S. R. and Malik, A. K. (2009). Two warehouses model with inflation induced demand under the credit period. International Journal of Applied Mathematical Analysis and Applications, 4(1), 59-70.
[29]. Singh, Y., Malik, A. K., Kumar, S., (2014). An inflation induced stock-dependent demand inventory model with permissible delay in payment. International Journal of Computer Applications, 96(25), 14-18.
[30]. Tiwari, S., Cardenas-Barron, L.E., Khanna, A., Jaggi, C.K. (2016) Impact of trade credit and inflation on retailer's ordering policies for non-instantaneous deteriorating items in a two-warehouse environment, International Journal of Production Economics, 176, 154-169.
[31]. Vandana, Sharma, B.K. (2016) An EOQ model for retailer's partial permissible delay in payment linked to order quantity with shortages, Mathematics and Computers in Simulation, 125, 99-112.
[32]. Vashisth, V., Tomar, A., Chandra, S., Malik, A. K. (2016). A trade credit inventory model with multivariate demand for non-instantaneous decaying products. Indian Journal of Science and Technology, 9(15), 1-6.
[33]. Yang H. L., Teng J. T. and Chern M. S. (2001) Deterministic inventory lot-size models under inflation with shortages and deterioration for fluctuation demand, Naval Research Logistics 48(2), 144-158.
[34]. Yang H.L. (2004) "Two-warehouse inventory models for deteriorating items with shortages under inflation", European Journal of Operational Research 157(2), 344-356.
[35]. Yang, H.L., Teng, J.T., Chern, M.S. (2010) An inventory model under inflation for deteriorating items with stock-dependent consumption rate and partial backlogging shortages, International Journal of Production Economics 123, 8-19.
[36]. Yang, H.L., Chang, C.T. (2013) A two-warehouse partial backlogging inventory model for deteriorating items with permissible delay in payment under inflation, Applied Mathematical Modelling, 37(5), 2717-2726.